Importance of Non-Linear Couplings in Fusion Barrier Distributions and Mean Angular Momenta

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Abstract

The effects of higher order coupling of surface vibrations to the relative motion on heavy-ion fusion reactions at near-barrier energies are investigated. The coupled channels equations are solved to all orders, and also in the linear and the quadratic coupling approximations. It is shown that the shape of fusion barrier distributions and the energy dependence of the average angular momentum of the compound nucleus can significantly change when the higher order couplings are included. The role of octupole vibrational excitation of ^{16}O in the ^{16}O + ^{144}Sm fusion reaction is also discussed using the all order coupled-channels equations.

1. Introduction

The analysis of the fusion process in terms of the barrier distribution has generated renewed interest in heavy-ion collisions at energies below and near the Coulomb barrier[1]. In a simple eigenchannel approach, couplings of the relative motion of the colliding nuclei to nuclear intrinsic degrees of freedom result in the single fusion barrier being replaced by a distribution of potential barriers. A method of extracting barrier distributions directly from fusion excitation functions was proposed[2], and stimulated precise measurements of the fusion cross sections for several systems. The analysis of the barrier distribution clearly shows the effects of couplings to several nuclear intrinsic motions in a way much more apparent than in the fusion excitation function itself[1].

Theoretically the standard way to address the effects of the coupling between the relative motion and the intrinsic degrees of freedom is to solve the coupled-channels equations, including all the relevant channels. Most of the coupled channels calculations performed so far use the linear coupling approximation, where the coupling potential is expanded in powers of the deformation parameter, keeping only the linear term. Whilst this approach reproduces the experimental data of fusion cross sections for very asymmetric systems,

it does not explain the data for heavier and nearly symmetric systems[3]. Thus, it is of interest to examine the validity of one of the main approximations in these calculations, namely the linear coupling approximation, and see whether the effects of non-linear coupling improve the agreement between data and the theoretical calculations for such systems. Even in asymmetric systems, the non-linear couplings might be important to reproduce precisely measured data.

In this contribution, we report the results of the coupled channels calculations which include the couplings to all orders[4, 5]. The results of these calculations for fusion cross sections, average angular momenta and barrier distributions are compared with those using the linear and the quadratic coupling approximations. It is seen that the linear coupling approximation is not valid even in systems where the coupling is weak, and that higher order couplings strongly influence the calculated barrier distributions.

2. Coupled channels equations in all orders

Consider the problem where the relative motion between colliding nuclei couples to a vibrational mode of excitation of the target nucleus. For simplicity excitations of the projectile are not considered in this section. It is straightforward to extend the formulae to the case where many different vibrational modes are present and where projectile excitations also occur. For heavy ion fusion reactions, to a good approximation one can replace the angular momentum of the relative motion in each channel by the total angular momentum J[6]. This approximation, often referred to as no-Coriolis approximation, will be used throughout this paper. The coupled channels equations then read

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{J(J+1)\hbar^2}{2\mu r^2}+\frac{Z_PZ_Te^2}{r}+n\hbar\omega-E_{cm}\right]\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0, \quad (1)$$

where r is the radial component of the coordinate of the relative motion, μ the reduced mass, E_{cm} the bombarding energy in the center of mass frame and $\hbar\omega$ is the excitation energy of the vibrational phonon. V_{nm} are the coupling matrix elements, which in the collective model consist of Coulomb and nuclear components.

For the Coulomb coupling, it has been shown that inclusion of up to the first order term is sufficient[4]. The matrix elements of the Coulomb coupling in Eq. (1) thus read

$$V_{nm}^{(C)}(r)=\frac{3}{2\lambda+1}Z_PZ_Te^2\frac{R_T^\lambda}{r^{\lambda+1}}\sqrt{\frac{2\lambda+1}{4\pi}}\alpha_0(\sqrt{n}\delta_{n,m+1}+\sqrt{n+1}\delta_{n,m-1}), \quad (2)$$

where α_0 is the amplitude of the zero point motion. It is related to the deformation parameter β_λ by $\alpha_0=\beta_\lambda/\sqrt{2\lambda+1}$[7].

In the collective model, the nuclear interaction is assumed to be a function of the separation distance between the vibrating surfaces of the colliding nuclei. It is conventionally taken as

$$V^{(N)}(r,\alpha_{\lambda 0})=-\frac{V_0}{1+\exp[(r-R_P-R_T-\sqrt{\frac{2\lambda+1}{4\pi}}R_T\alpha_{\lambda 0})/a]}, \quad (3)$$

where the surface coordinates $\alpha_{\lambda\mu}$ are related to the phonon creation and annihilation operators by $\alpha_{\lambda\mu}=\alpha_0(a_{\lambda\mu}^\dagger+(-)^\mu a_{\lambda-\mu})$. Volume conservation introduces a small term

which is non-linear with respect to the deformation parameter $\alpha_{\lambda 0}$ in the denominator of the above Eq.(3). This is ignored for simplicity in the present study.

The conventional nuclear coupling form factor in the linear coupling approximation is obtained by expanding Eq.(3) with respect to $\alpha_{\lambda 0}$ and keeping only the linear term. It is given by

$$V_{nm}^{(N)}(r) = V_N(r)\delta_{n,m} - \sqrt{\frac{2\lambda+1}{4\pi}}\alpha_0 \frac{dV_N(r)}{dr}(\sqrt{n}\delta_{n,m+1} + \sqrt{n+1}\delta_{n,m-1}), \tag{4}$$

where $V_N(r) = -V_0/[1+\exp((r-R_p-R_T)/a)]$ is the nuclear potential in the entrance channel.

In order to take into account the effects of the couplings to all order, we evaluate the nuclear coupling matrix elements without introducing the expansion. Denoting the eigenvalue of $\alpha_{\lambda 0}$ by x, the matrix elements of the nuclear coupling then read

$$V_{nm}^{(N)}(r) = \int_{-\infty}^{\infty} dx u_n^*(x) u_m(x) \frac{-V_0}{1+\exp[(r-R_P-R_T-\sqrt{\frac{2\lambda+1}{4\pi}}R_T x)/a]} \quad . \tag{5}$$

Here $u_n(x)$ is the eigen function of the n -th excited state of the harmonic oscillator and is given by

$$u_n(x) = \frac{1}{2^n n!}\frac{1}{\sqrt[4]{2\pi\alpha_0^2}} H_n\left(\frac{x}{\sqrt{2}\alpha_0}\right) e^{-x^2/4\alpha_0^2}, \tag{6}$$

where $H_n(x)$ is the Hermite polinomial with rank n.

3. Validity of the linear coupling approximation

3.1. Nearly Symmetric Systems

We now present the results of our calculations of fusion cross sections, average angular momenta of the compound nucleus, and fusion barrier distributions. We first discuss heavy nearly symmetric systems. We analyse in particular ^{64}Ni + ^{96}Zr reactions which are typical examples where the conventional coupled channels calculations with the linear coupling approximation fail to reproduce the fusion cross sections and average angular momentum data[8]. Our aim is to investigate whether the failure is due to the linear coupling approximation by performing linear, quadratic and full coupling calculations.

We take into account the couplings up to two phonon states of the quadrupole surface vibrations of ^{64}Ni, and of the octupole vibration of ^{96}Zr. We also take their mutual excitations into account. We ignore the effects of transfer reactions, because it has been reported in Ref.[8] that they have only small effects on the fusion cross sections and the average angular momenta in this reaction. The excitation function of the fusion cross section for this reaction obtained by numerically solving the coupled channels equations is compared with the experimental data in Fig. 1 (upper panel). The experimental data, taken from Ref. [8], consist only of the evaporation residue cross sections, and do not include fission following fusion. The dotted line is the results in the one dimensional

potential model, *i.e.* without the effects of channel coupling. The dot-dashed line is the results of the coupled channels calculations when the linear coupling approximation is used. They considerably underestimate the fusion cross sections at sub-barrier energies. The situation is slightly improved when the quadratic coupling approximation[9] is used, *i.e.* when the nuclear coupling potential up to the second order of the deformation parameter, is included (dashed line). However, there still remain considerable discrepancies between the experimental data and the results of the coupled channels calculations. When we include couplings to all order, we get the solid line, which agree very well with the experimental data. Dramatic effects of the higher order couplings on fusion cross sections are observed, especially at low energies.

The lower panel in Fig. 1 compares the results of our calculations of the average angular momentum of the compound nucleus with the experimental data as a function of the bombarding energy. We again observe that the experimental data are much better reproduced by taking the effects of couplings to all orders into account. We thus conclude that coupling to all orders are essential to simultaneously reproduce the fusion cross sections and the average angular momentum data for heavy (nearly) symmetric systems. This is in agreement with the calculations required to fit the barrier distribution for ^{58}Ni + ^{60}Ni reaction[10].

3.2. Very Asymmetric Systems

We next consider the effects of higher order couplings for very asymmetric systems where the product of the charges $Z_P Z_T$ is relatively small. For such systems, the coupled channels calculations in the linear coupling approximation have achieved reasonable success in reproducing fusion excitation functions. However, no study has been performed to see whether the effects of higher order couplings on the angular momentum distribution of the compound nucleus and on the barrier distributions are small. In this subsection we re-analyse the experimental data for the ^{16}O + ^{144}Sm reaction, for which the effects of couplings to phonon states on fusion barrier distributions were shown experimentally for the first time[1].

The fusion barrier distribution for this system is intimately related to the octupole vibration of ^{144}Sm, and that the quadrupole vibration plays only a minor role[1]. Accordingly, we ignore the effects of the couplings to the quadrupole phonon states of ^{144}Sm and include only the single octupole phonon state at 1.81 MeV. For simplicity in the calculations we ignore excitation of the projectile. These effects will be discussed in Sec. 4.

The upper panel of Fig. 2 shows the experimental fusion excitation function taken from Ref. [1], and the theoretical calculations. The meaning of each line is the same as in Fig.1. We observe that the agreement of the theory and experiment appears to be improved only slightly by the inclusion of coupling to all orders. The barrier distribution, on the other hand, reveals significant changes due to the higher order couplings (see the lower panel of Fig. 2). Comparing the results of the linear coupling approximation (the dot-dashed line) with those of the all order coupling (the solid line), one observes that the higher order couplings transfer some strength from the lower barrier to the higher barrier, and at the same time lower the peak position of both barriers.

Figure 3 shows the coupling matrix element between the ground state and the one phonon state $V_{01}(r)$ as a function of the inter nuclear separation distance r. The dotted line is the coupling matrix element in the linear coupling approximation, while the solid line includes the coupling to all orders. One can see that the linear coupling approximation underestimates the coupling strength at the barrier position of the uncoupled barrier around $r=$ 10.8 fm. On the other hand, it overestimates the coupling strength in the inner region around r=8.5 fm. As we will see in the next section, the latter fact is important in discussing the effects of the projectile excitation.

The effects of higher order couplings become more significant when there exist more than two channels. In order to demonstrate this, we show in fig. 4 the barrier distributions for the ^{16}O + ^{144}Sm reaction, where the double octupole phonon excitations are allowed in the harmonic limit. One observes dramatic effects of higher order couplings on the fusion barrier distribution. It is thus clear that high precision measurements should be analysed using all order couplings even when coupling is weak as a result of the small charge product. The detailed studies on the effects of double phonon couplings including anharmonic effects in this reaction is reported separately [11, 12].

4. Role of projectile excitation in fusion

Let us now discuss the role of the octupole vibrational excitation of ^{16}O in the ^{16}O + ^{144}Sm fusion reactions[13]. Contradictory conclusions have been reported regarding the role of projectile excitation in the fusion reactions between ^{16}O and samarium isotopes. Calculations of the fusion cross-section for the ^{16}O + ^{154}Sm reactions in ref. [14] indicated the importance of the excitation of ^{16}O. In marked contrast, no specific features appear in the measured barrier distribution for the ^{16}O + ^{144}Sm reactions which can be associated with the excitation of ^{16}O; rather, it was argued in Ref. [1] that a good theoretical representation of the experimental fusion barrier distribution is destroyed when the projectile excitation is included.

Both of these conclusions are based on the comparison of the experimental data with the results of simplified coupled-channels calculations, where the linear coupling approximation is used to describe the vibrational excitation of the projectile. In the previous section, we have shown that the linear coupling approximation is not valid even in systems with weak coupling, and that higher order couplings strongly influence the fusion barrier distribution. The coupling to the octupole vibrational state of ^{16}O is strong because of the large deformation parameter indicated by the strong E3 transition. It is therefore very likely that the fusion barrier distribution calculated in the simplified coupled–channel codes using the linear coupling approximation does not represent the true fusion barrier distribution.

In order to test the validity of the simplified coupled-channels calculations, here we first calculate the excitation function of the fusion cross section and the fusion barrier distribution in the linear coupling approximation. The results are shown in Fig. 5. The dotted line is the results when ^{16}O is treated to be inert. This calculation well reproduces the features of the experimental barrier distribution. The results of calculations including the excitation of the lowest-lying octuple state of ^{16}O are shown by the solid line. Though the experimental barrier distribution around the lower energy peak at ($\sim$ 60 MeV) is

reproduced, significant strength is missing around the higher energy peak near 65 MeV. A similar discrepancy between theory and experimental data was encountered in Ref. [1], where calculations were performed using a modified version of the CCFUS code (the long–dashed line). Clearly both calculations which treat the octupole excitation of ^{16}O in the linear coupling approximation fail to reproduce the experimental barrier distribution.

The results of coupled-channels calculations, where the couplings to the octupole vibrations of both ^{16}O and ^{144}Sm are treated to all orders, are shown in figure 6. It is remarkable that these calculations re-establish the double-peaked structure seen in the experimental data, which was missing in the linear coupling calculations. Indeed, the barrier distribution obtained by including the coupling to the octupole vibration of ^{16}O to all order looks very similar to that obtained by totally ignoring it, apart from a shift in energy. A shift of 2 MeV (dashed line) of the former is required for the two calculated distributions to coincide. This shift is consistent with the general conclusion that the main effect of the coupling to an inelastic channel whose excitation energy is larger than the curvature of the bare fusion barrier, i.e. an adiabatic coupling, is to introduce a static potential shift[15], and hence, the shape of the barrier distribution does not change unless the coupling is very strong and the coupling form factor itself has a strong radial dependence. The effects of these excitations can then be included in the 'bare' potential in the coupled-channels calculations. Thus for ^{16}O + ^{144}Sm, where potential parameters for the calculations are obtained from a fit to the high energy data, the effects of octupole vibration of ^{16}O are already included. The explicit inclusion of the coupling to octupole vibration then leads to double counting which manifests itself by introducing an additional shift in the barrier (or barrier distribution) as observed earlier.

5. Summary

We have shown that higher order couplings to nuclear surface vibrations play an important role in heavy ion fusion reactions. The inclusion of the coupling to all orders in coupled-channels calculations was shown to be crucial to reproduce the experimental fusion cross sections and the average angular momenta for heavy symmetric systems. We performed coupled-channels calculations also for the ^{16}O + ^{144}Sm reactions as an example of very asymmetric systems where the coupling is weaker. It was found that higher order couplings to the vibrational states of the target nucleus result in a non-negligible enhancement of the fusion cross sections and a significant modification of barrier distributions even in very asymmetric systems. Our studies warn that spurious conclusions could be reached regarding the nature of couplings if high quality experimental data are compared with simplified calculations in the first order approximation. We also studied the role of projectle excitation in the ^{16}O + ^{144}Sm fusion reactions. Using the calculations with full order coupling we have shown that the major effect of the excitation of the octupole state at 6.1 MeV of ^{16}O is to renormalize the static potential barrier without significantly modifying the shape of the barrier distribution.

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Figure Captions

Fig.1: Excitation function of the fusion cross section (upper panel) and the average angular momentum of the compound nucleus (lower panel) for the ^{64}Ni + ^{92}Zr reactions.
Fig.2: Excitation function of the fusion cross section (upper panel) and the fusion barrier distribution (lower panel) for the ^{16}O + ^{144}Sm reactions. In the coupled channels calculations, the projectile is assumed to be inert, while the single octupole phonon state of the target nucleus is taken into account.
Fig.3: The coupling matrix element between the ground state and the one phonon state for the ^{16}O + ^{144}Sm reaction as a function of the separation distance between the projectile and target..
Fig.4: Same as the lower panel of fig. 2, but for the case where the double octupole phonon excitations of ^{144}Sm are included in the harmonic limit.
Fig.5: Effects of the projectile excitation on the ^{16}O + ^{144}Sm fusion reactions. The linear coupling approximation is used in the coupled-channels calculations. In all calculations, the effects of the octupole vibration of ^{144}Sm are taken into account.
Fig.6: Same as Fig.5, but for the case where the coupled-channels calculations have been performed to all order coupling.

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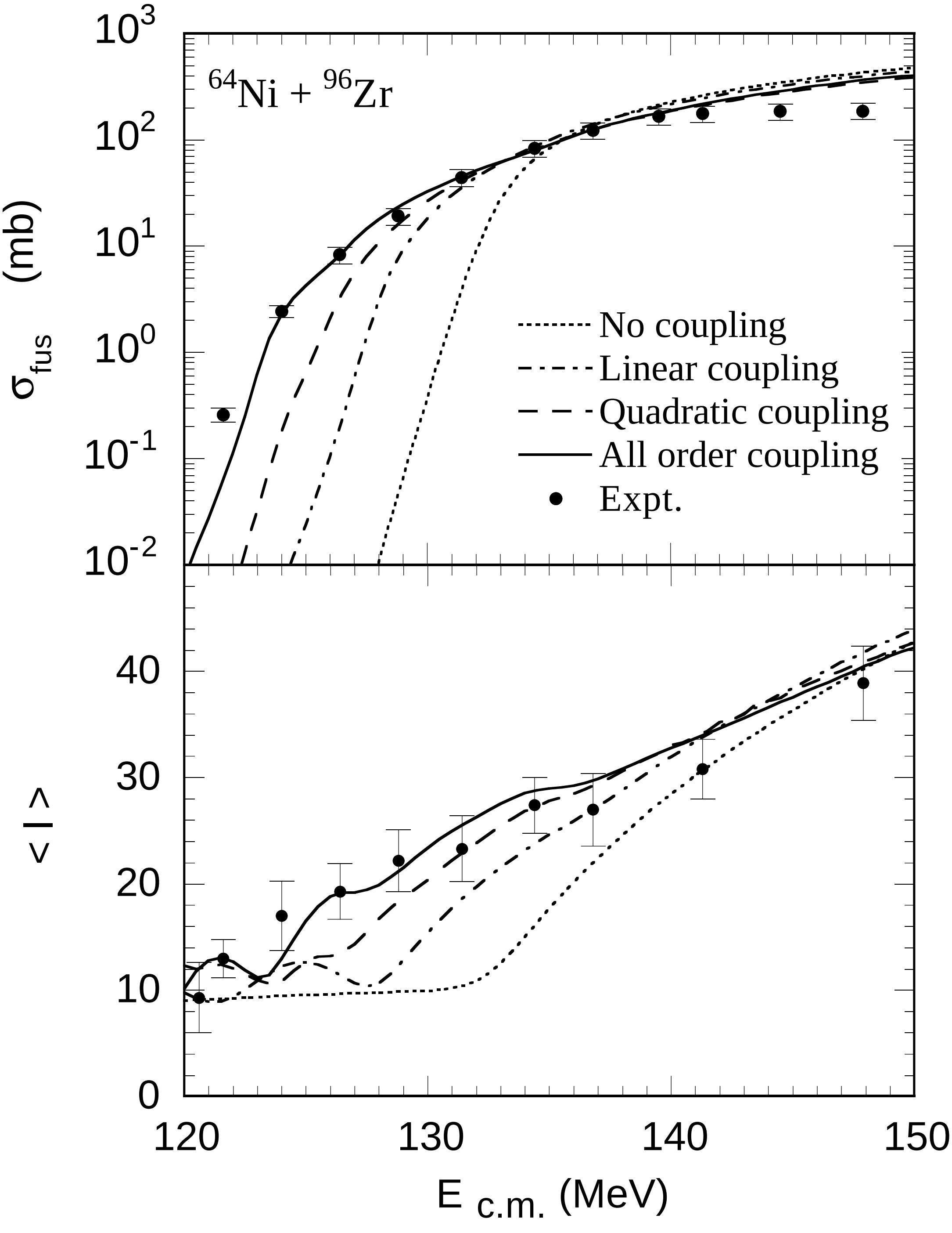
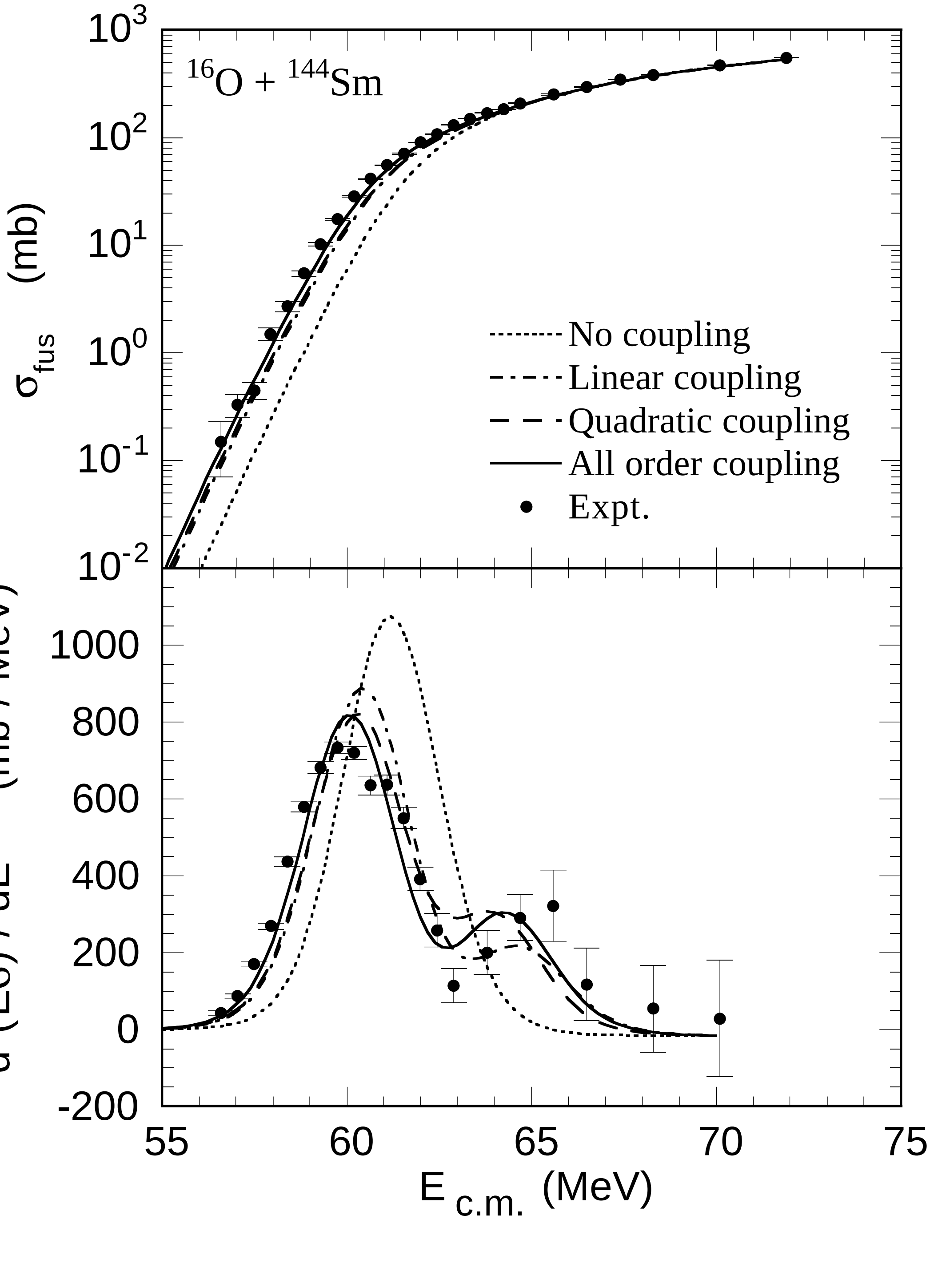
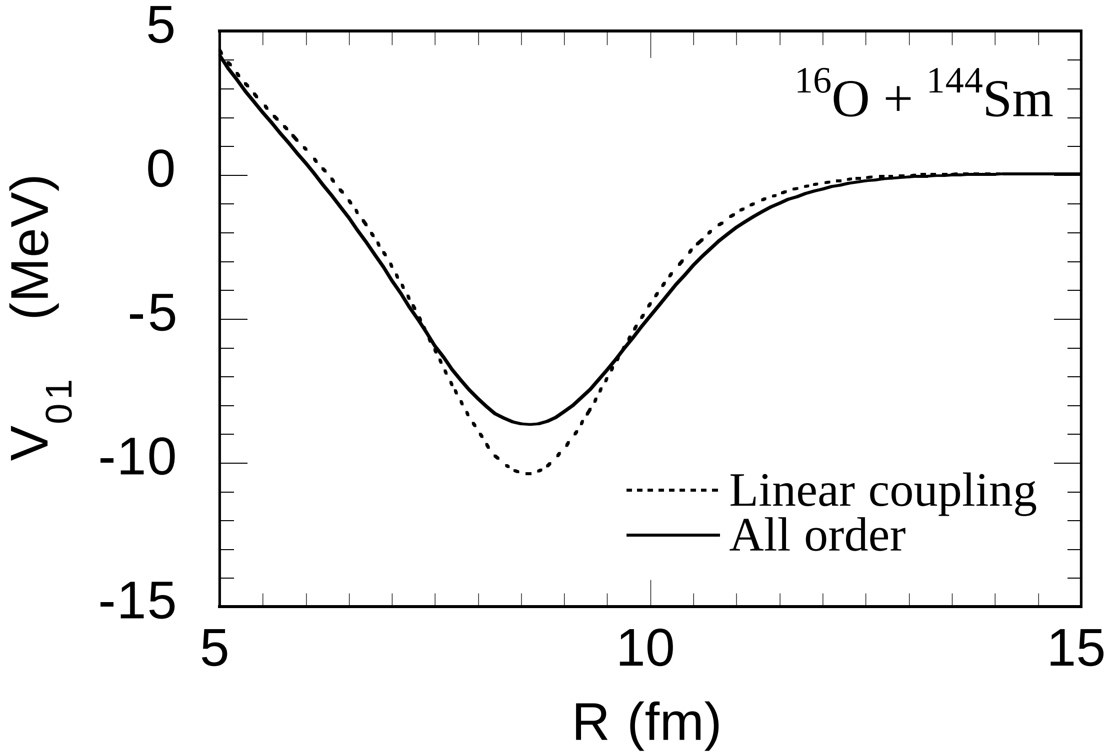


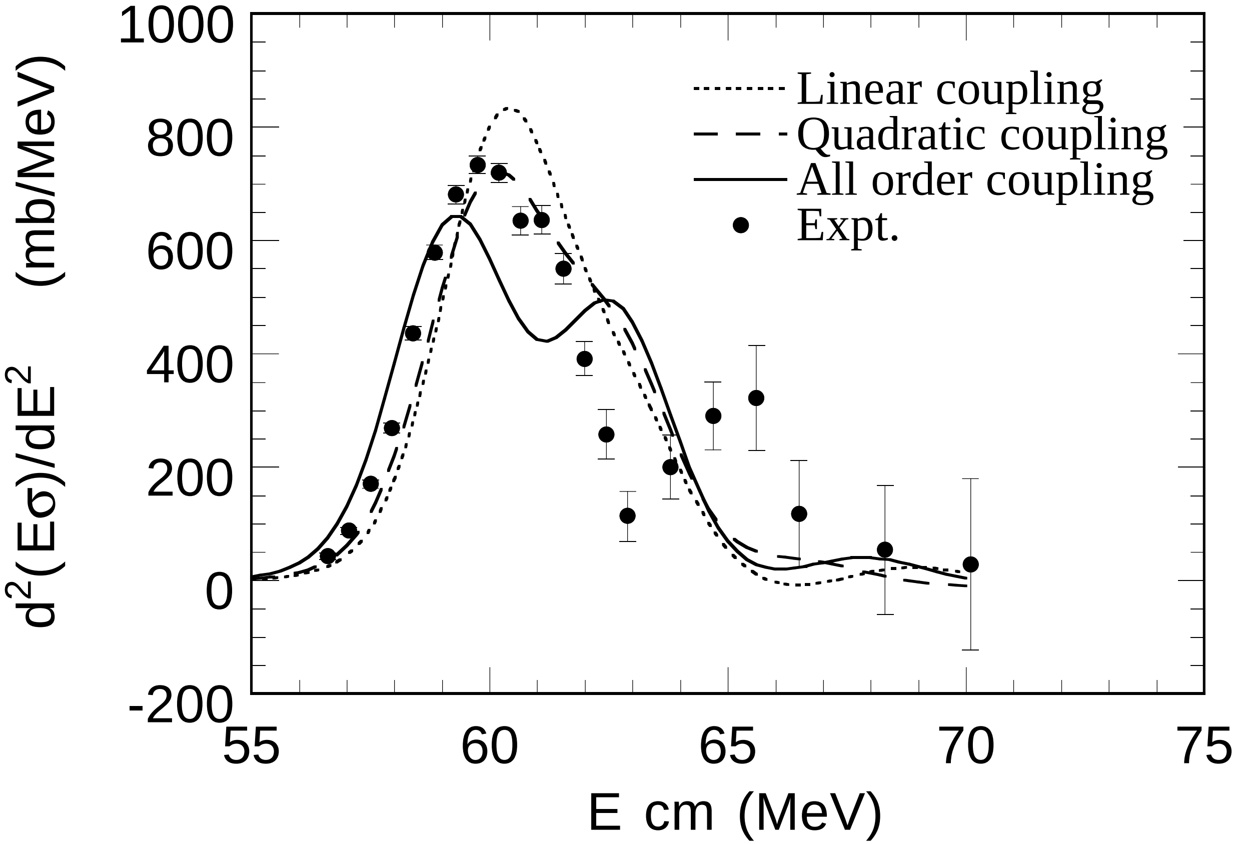
^{64}Ni + ^{96}Zr
No coupling
Linear coupling
Quadratic coupling
All order coupling
Expt.
σ_{fus} (mb)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
< l >
0
10
20
30
40
120
130
140
150
E c.m. (MeV)



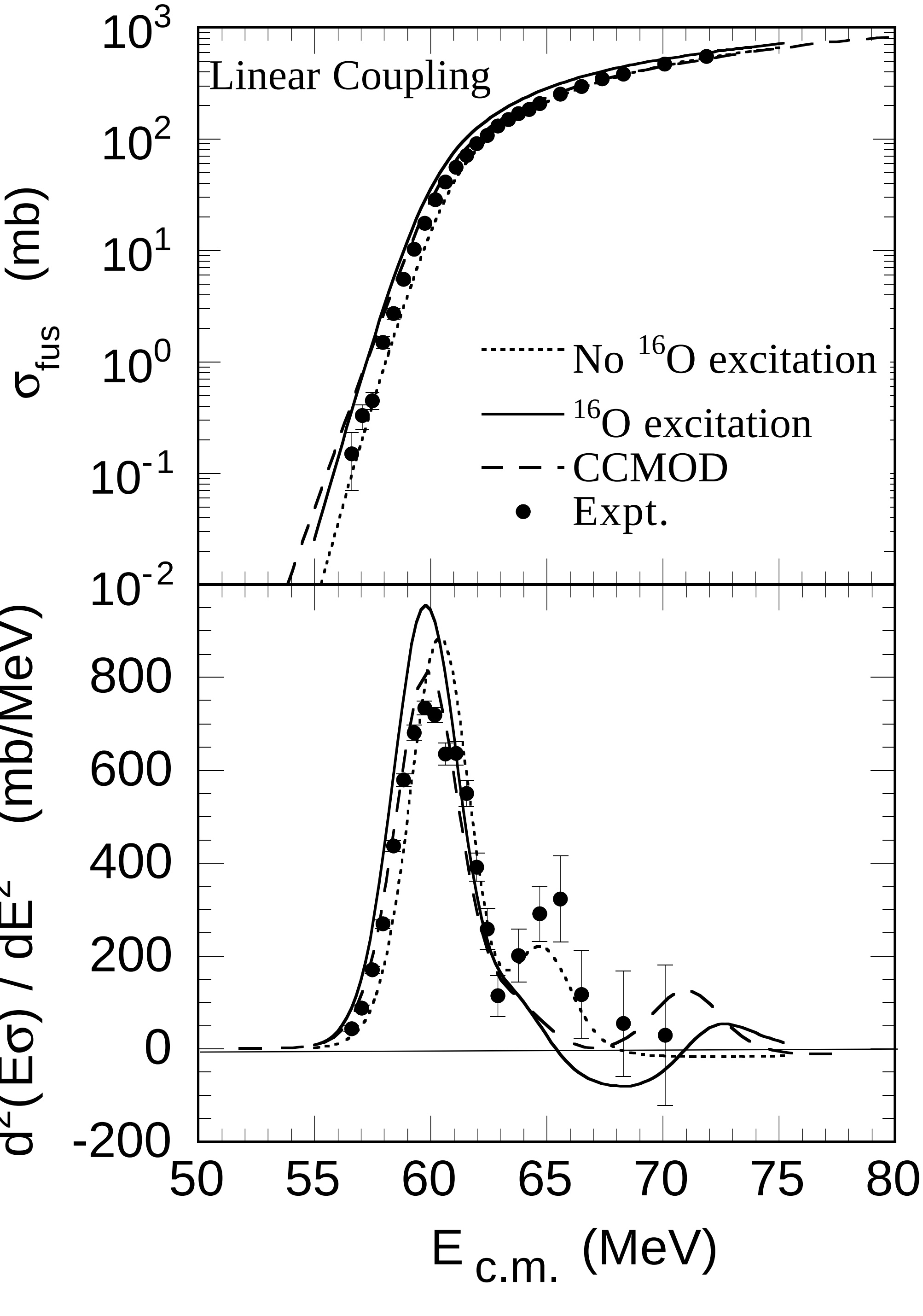
^{16}O + ^{144}Sm
No coupling
Linear coupling
Quadratic coupling
All order coupling
Expt.
σ_{fus} (mb)
$d^2(E\sigma) / dE^2$ (mb / MeV)
$E_{c.m.}$ (MeV)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
-200
0
200
400
600
800
1000
55
60
65
70
75



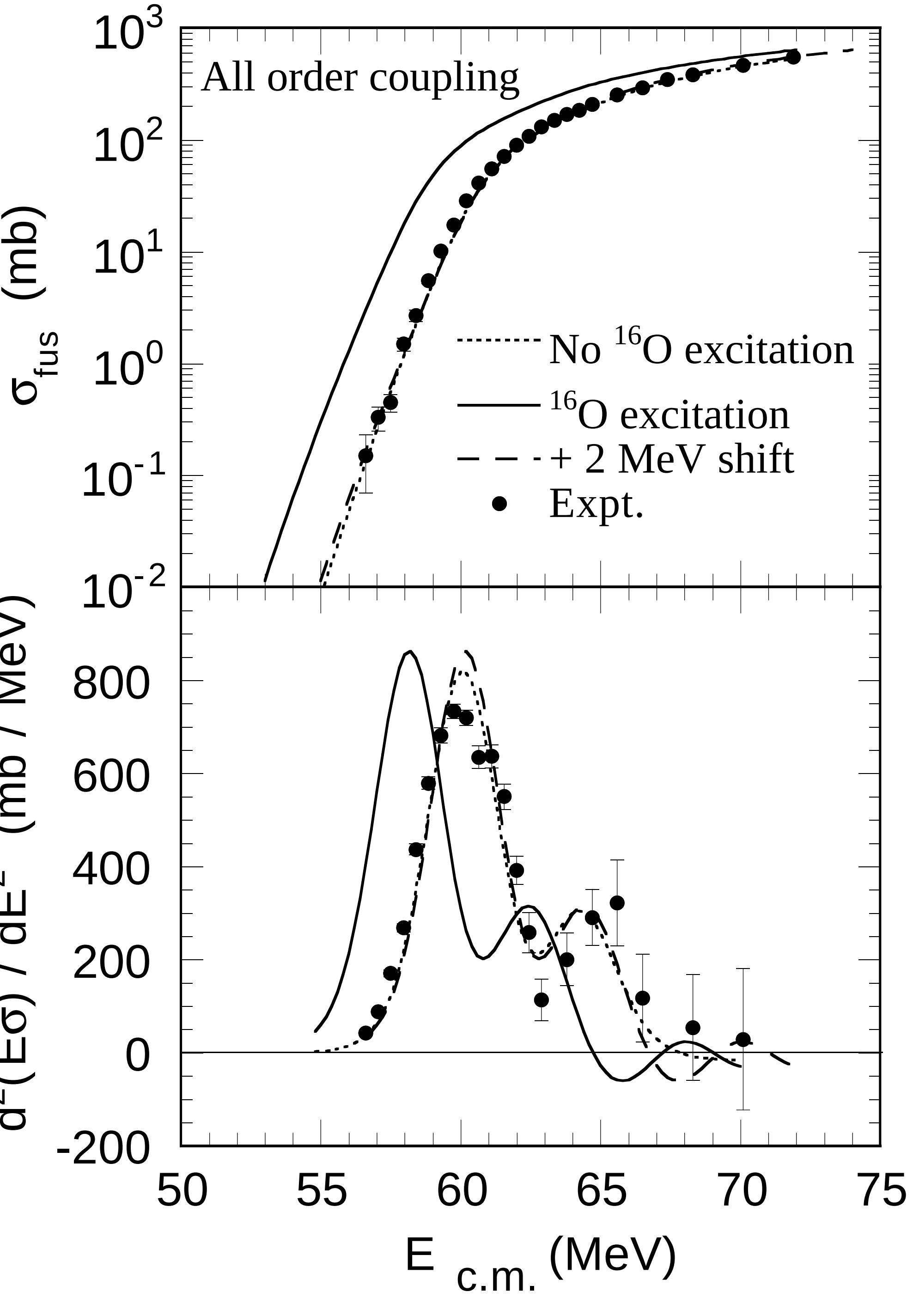
^{16}O + ^{144}Sm
Linear coupling
All order
V_{01} (MeV)
R (fm)
5
0
-5
-10
-15
5
10
15



1000
800
600
400
200
0
-200
55
60
65
70
75
E cm (MeV)
d2(Eσ)/dE2 (mb/MeV)
Linear coupling
Quadratic coupling
All order coupling
Expt.



Linear Coupling
No ^{16}O excitation
^{16}O excitation
CCMOD
Expt.
σ_{fus} (mb)
$d^2(E\sigma) / dE^2$ (mb/MeV)
$E_{c.m.}$ (MeV)



All order coupling
No ^{16}O excitation
^{16}O excitation
+ 2 MeV shift
Expt.
σ_{fus} (mb)
$d^2(E\sigma) / dE^2$ (mb / MeV)
$E_{c.m.}$ (MeV)